wireless network coding in two-way relay channels

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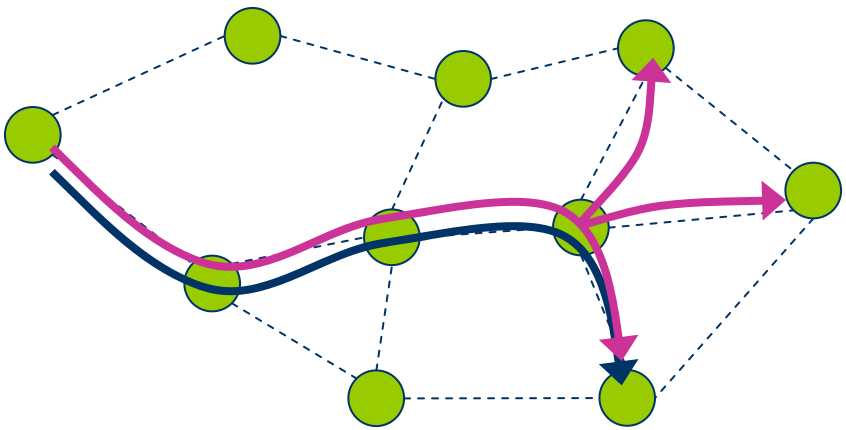
talk outline

- introduction to network coding
- wireless network coding
- 3-step schemes
- 2-step schemes
- numerical results
- conclusion

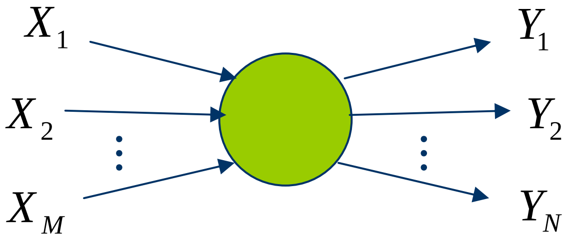
- credits
  - joint work with hiroyuki yomo

- communication networks
  - two nodes communicate if a random process at the first node is reproduced at the second node
  - source and destination nodes



- types of connections
  - unicast, multicast

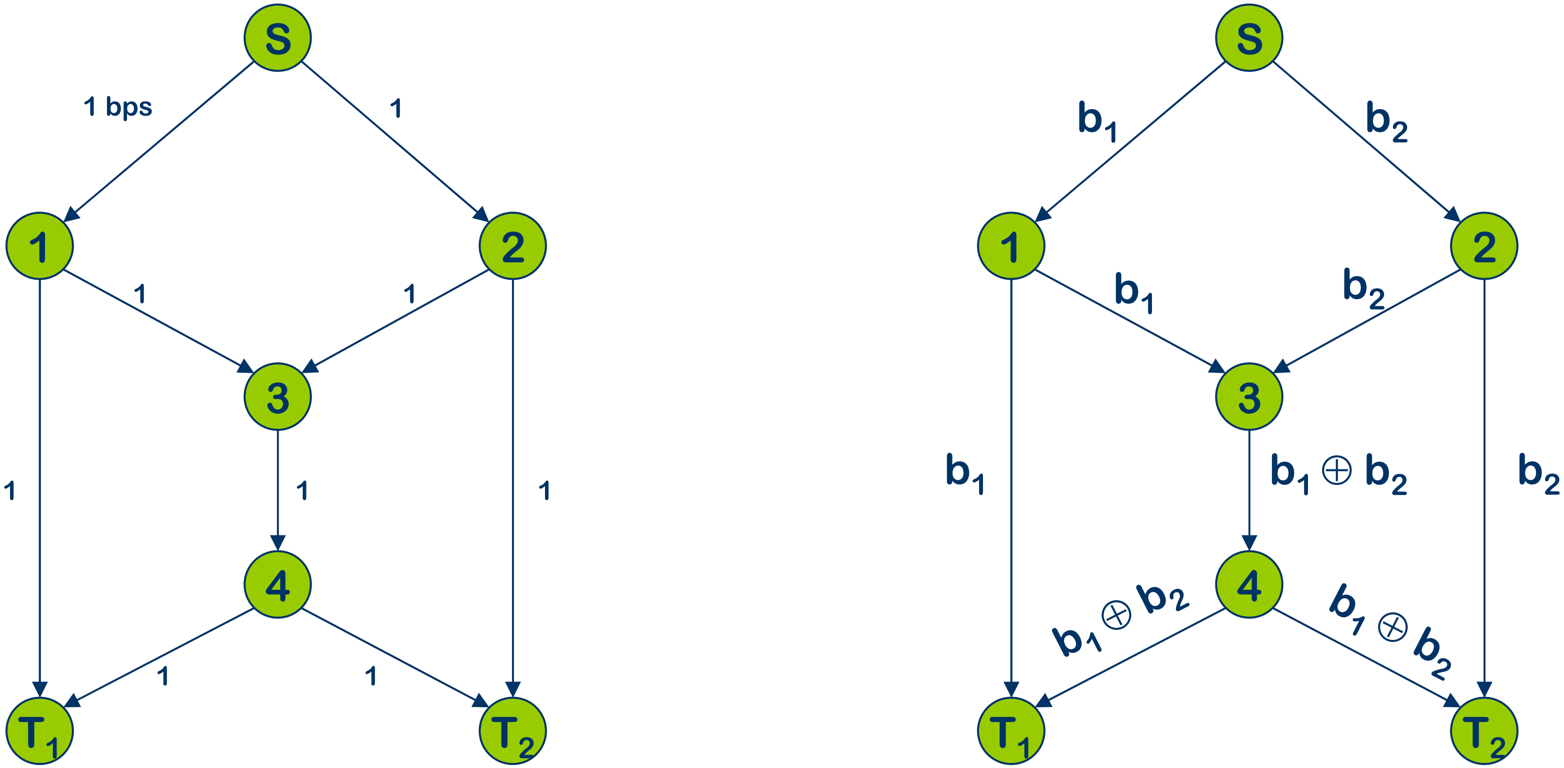
- the routing function treats the information flow as a commodity flow and finds a path from source to the destination
- traditionally, a routing node acts as a switch: each of the output flows must be identical to one of the input flows



- the network coding uses more general mapping

$$Y_n = f_n\left(X_1, X_2, \cdots X_M\right)$$

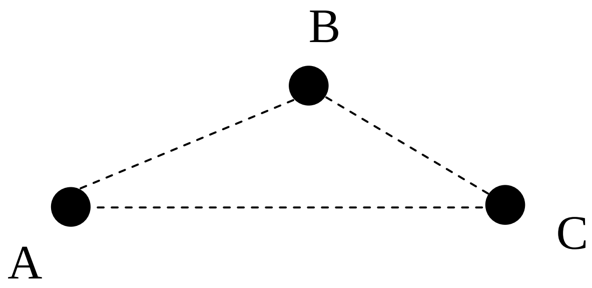
the basic example of network coding



R. Ahlswede, N. Cai, S. R. Li, and R. W. Yeung. “Network information flow”. IEEE Transactions on Information Theory, July 2000.

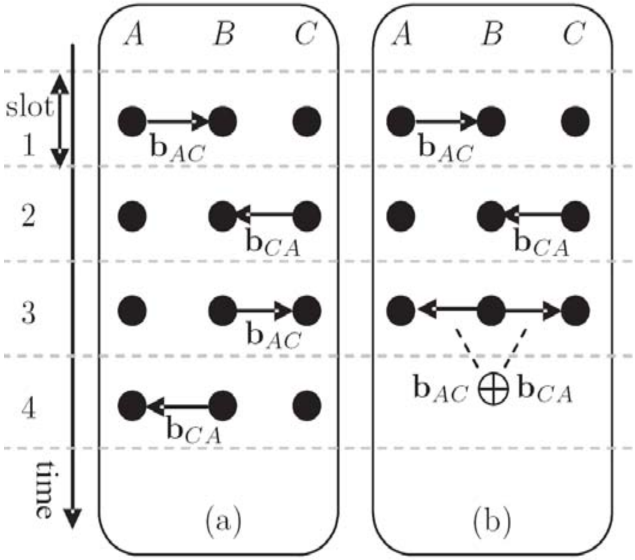
- takes advantage of the shared wireless medium

- application in two-way (bi-directional) relay channels



- A wants to send the packet D_{AC} to C
- C wants to send the packet D_{CA} to A
- B is a relay (helper) node
- all devices are half-duplex

two-way relaying with wireless network coding



two-way relaying with Decode-and-Forward (DF)

throughput amplification in absence of noise

the packet size is N bits, the slot duration is T_s

transmission rate over a link $R_s = \frac{N}{T_s}$

rate for the conventional relaying $\frac{2N}{4T_s} = \frac{R_s}{2}$

rate for relaying with network coding $\frac{2N}{3T_s} = \frac{2R_s}{3}$

- the basic observation is that broadcast has the same price as unicast in wireless networks

- throughput amplification for symmetric traffic

$$G_{DF} = \frac{\frac{2N}{3T_s} - \frac{2N}{4T_s}}{\frac{2N}{4T_s}} = \frac{\frac{2}{3}R_s - \frac{R_s}{2}}{\frac{R_s}{2}} = \frac{1}{3} = 33.33\%$$

- impact of traffic asymmetry

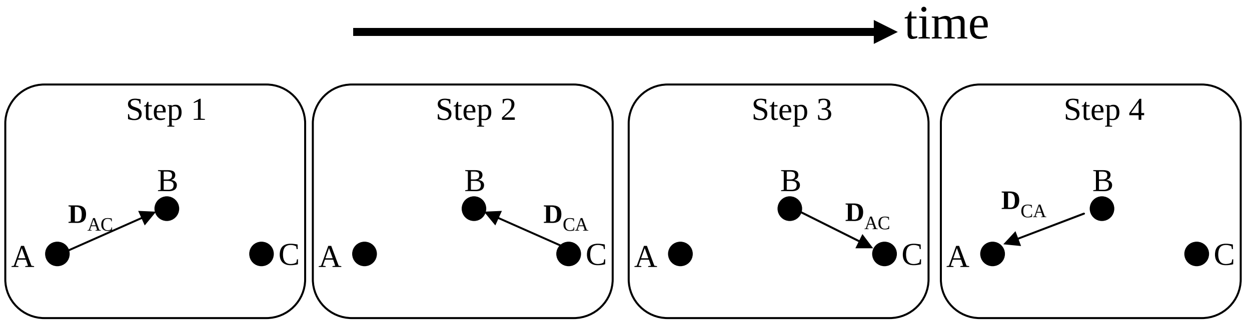
Asymmetry factor

$$K_A \leq K_C \qquad \alpha = \frac{K_A}{K_A + K_C}$$

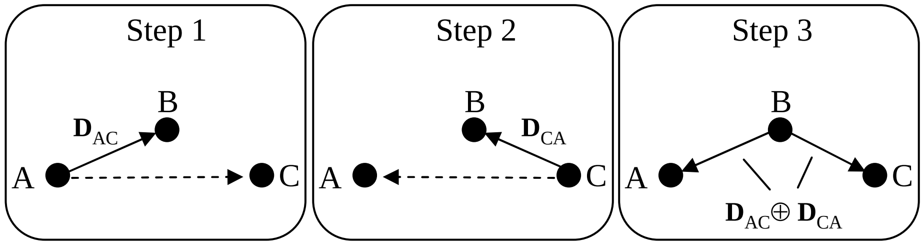
For $\alpha < 0.5$ there are not enough b_{AC} packets to be XOR-ed with the b_{CA} packets

$$G_{DF}(\alpha) = \frac{\alpha}{2-\alpha}$$

transmission schemes for two-way relaying channel

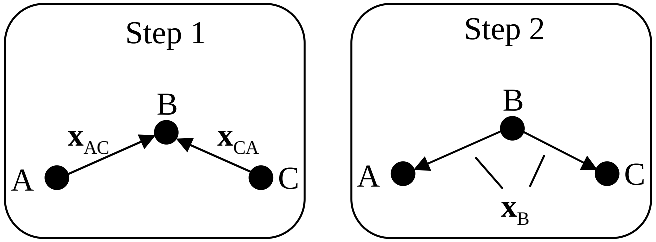


conventional relaying



decode-and-forward (DF)

- Larsson et al.
- Wu, Chou, Kung
- Popovski, Yomo
- Katti et al.



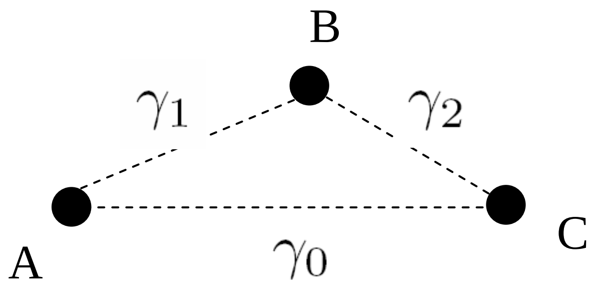
amplify-and-forward (AF)

- Larsson et al.
- Popovski, Yomo
- Rankov, Wittneben

denoise-and-forward (DNF)

- Popovski, Yomo
- Xiao et al.
- Zhang et al.

notations and definitions



- normalized bandwidth to 1 Hz
  - time measured in number of symbols

$$C(\gamma) = \log_2(1+\gamma) \ \text{[bit/s]}$$

$$y_V[m] = h_{UV} x_U[m] + z_V[m]$$

Gaussian

multiple-access channel

$$y_B[m] = h_1 x_A[m] + h_2 x_B[m] + z_B[m]$$

$$\gamma_1 > \gamma_0 \qquad \gamma_2 > \gamma_0$$

$$\gamma_2 \geq \gamma_1$$

two-way rate

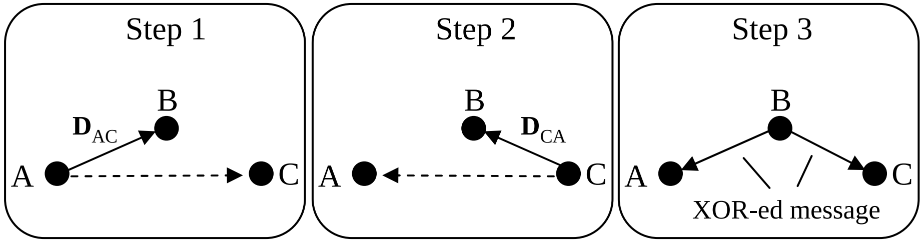
$$R_{A\leftrightarrow C} = \frac{|\mathbf{D}_{AC}| + |\mathbf{D}_{CA}|}{N} \ \text{[bits/s]}$$

problem definition

- find the maximal two-way rates that can be achieved by the different schemes when each individual link is represented by AWGN channel

- we put some operational constraints
  - in each round A and C transmit only fresh data
  - the broadcast strategy is potentially suboptimal (does not maximize the broadcast sum rate)

3-step scheme (1)



the problem is how to select

$$\theta$$

step 1	step 2	step 3
• A transmits D_{AC} at rate $C(\gamma_1)$ • B decodes D_{AC} • C retains the received signal for later decoding	• C transmits D_{CA} at $C(\gamma_2)$ • B decodes D_{CA} • A retains the received signal for later decoding	• B transmits to A and C

time

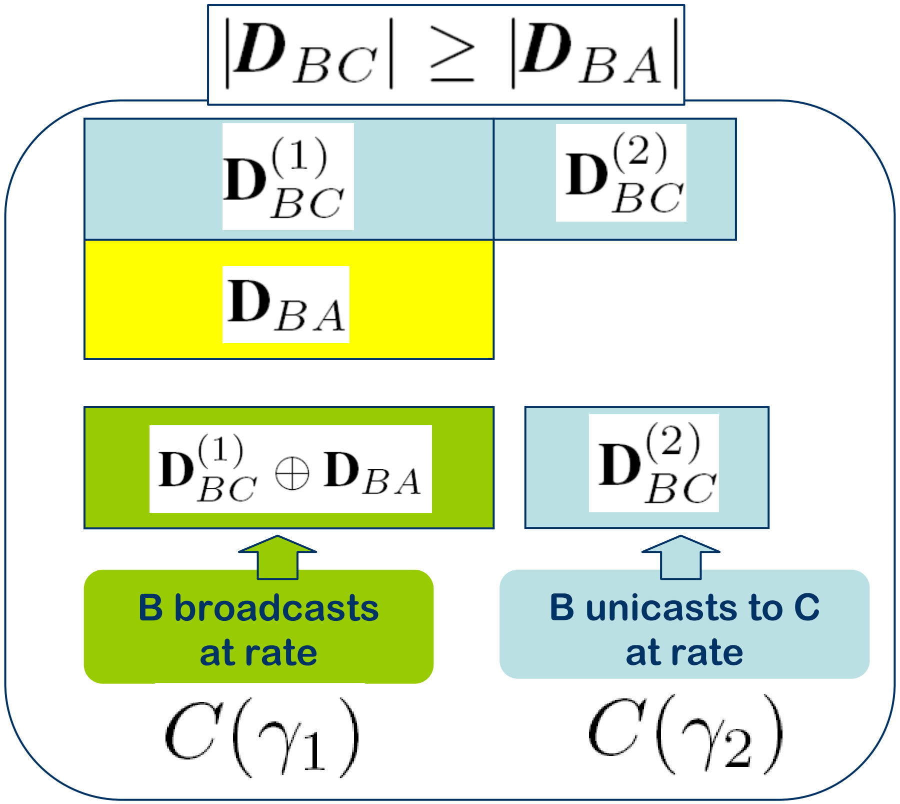
step 1: $N(1-\theta)$ symbols

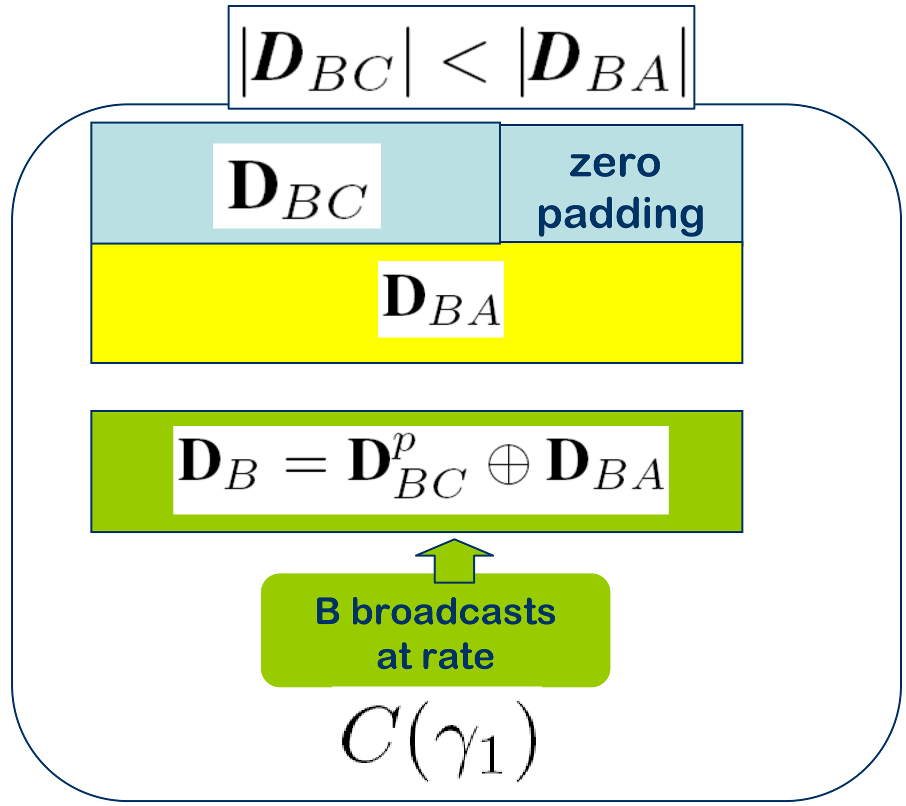
step 2: $N\theta$ symbols

step 3: number of symbols function of two previous steps

- in step 3 B should send
  - to C at least $|\mathbf{D}_{BC}| = N(1-\theta)\log_2\left[C(\gamma_1) - C(\gamma_0)\right])$ [bits]
  - to A at least $|\mathbf{D}_{BA}| = N\theta\log_2\left[C(\gamma_2) - C(\gamma_0)\right]$ [bits]

3-step scheme (2)





optimal choice of θ

$$\theta^* = \frac{C(\gamma_1) - C(\gamma_0)}{C(\gamma_1) + C(\gamma_2) - 2C(\gamma_0)} \leq \frac{1}{2}$$

maximized two-way rate

$$R_{DF}^* = C(\gamma_1)\frac{1 + \delta[C(\gamma_2) - C(\gamma_1)]}{1 + \delta[C(\gamma_2) - C(\gamma_0)]}$$

2-step schemes

- step 1 has always a duration of N symbols

- the key problem here is how to choose the transmission rates of A and C, so that at the end of 2 steps A receives D_{CA} and C receives D_{AC}

- B amplifies and broadcasts what is received in step 1

$$\mathbf{x}_B = \beta \mathbf{y}_B \qquad \beta = \sqrt{\frac{1}{|h_1|^2 + |h_2|^2 + N_0}}$$

- m-th symbol received at A

$$y_A[m] = \beta h_1 y_B[m] + z_A[m] =$$
$$\beta h_1^2 x_A[m] + \boxed{\beta h_1 h_2 x_C[m] + \beta h_1 z_B[m] + z_A[m]}$$

- by removing , A observes AWGN channel

$$\gamma_{C\rightarrow A}^{(AF)} = \frac{\gamma_1 \gamma_2}{2\gamma_1 + \gamma_2 + 1}$$

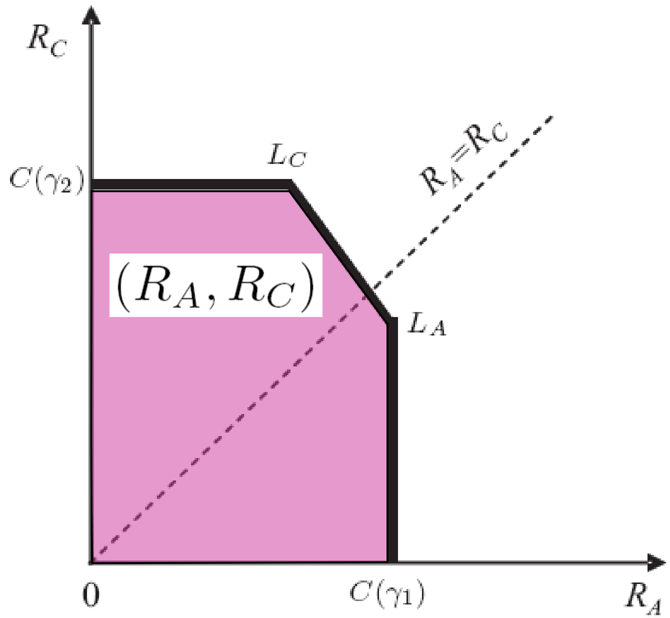
C sends for A at a rate $R_C = C\left(\gamma_{C\rightarrow A}^{(AF)}\right)$

A sends for C at a rate $R_A = C\left(\gamma_{A\rightarrow C}^{(AF)}\right)$

$$R_{AF} = \frac{R_A + R_C}{2}$$

joint decode-and-forward (JDF)

- **step 1:** A and C select the rates R_A and R_C to be jointly decodable at B
- **step 2:** B XORs the received packets and broadcasts them at a rate $C(\gamma_1)$



maximized two-way rate

$$R^*_{JDF} = C(\gamma_1)\frac{2C(\gamma_1+\gamma_2)}{2C(\gamma_1)+C(\gamma_1+\gamma_2)}$$
$$\text{if } \gamma_1 \le \gamma_2 \le \gamma_1 + \gamma_1^2$$

$$R^*_{JDF} = C(\gamma_1)$$
$$\text{if } \gamma_2 > \gamma_1 + \gamma_1^2$$

denoise-and-forward (DNF)

- **question:** can the relay "clean up" the noise without decoding the signals?
- **yes**, for which we have introduced the **denoise-and-forward (DNF)** scheme

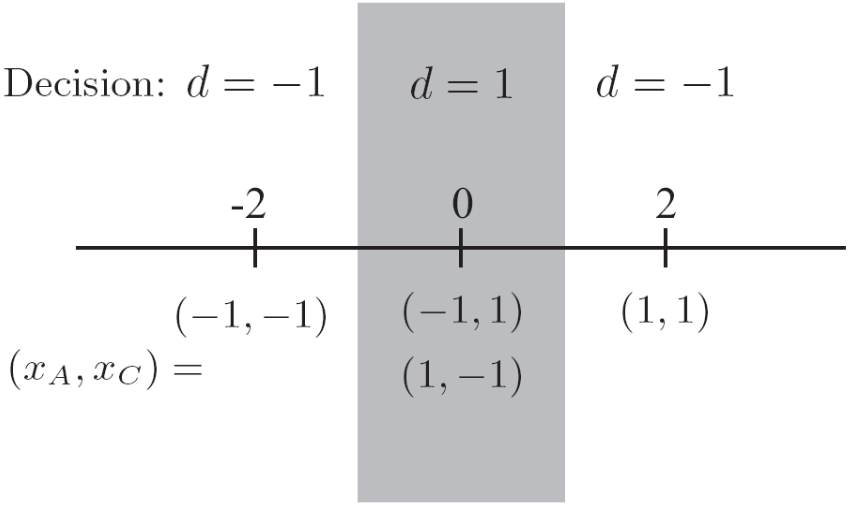
Example of DNF

Let both A and C use BPSK

$$x_A[m], x_C[m] \in \{-1, 1\}$$

and let

$$y_B[m] = x_A[m] + x_C[m] + z_B[m]$$

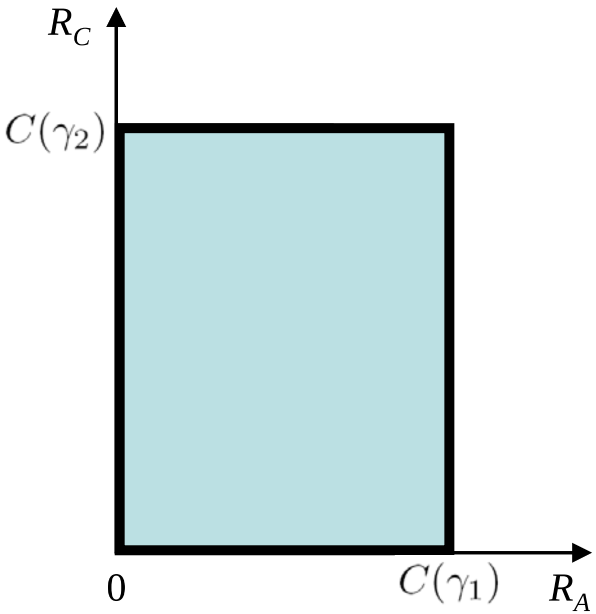


generalization of the DNF mechanism

$$\mathbf{y}_B = h_{AB}\mathbf{x}_A + h_{CB}\mathbf{x}_C + \mathbf{z}_B$$

– for each $\mathbf{y}_B$ there is set of K codeword pairs that are likely candidates to produce that output at B

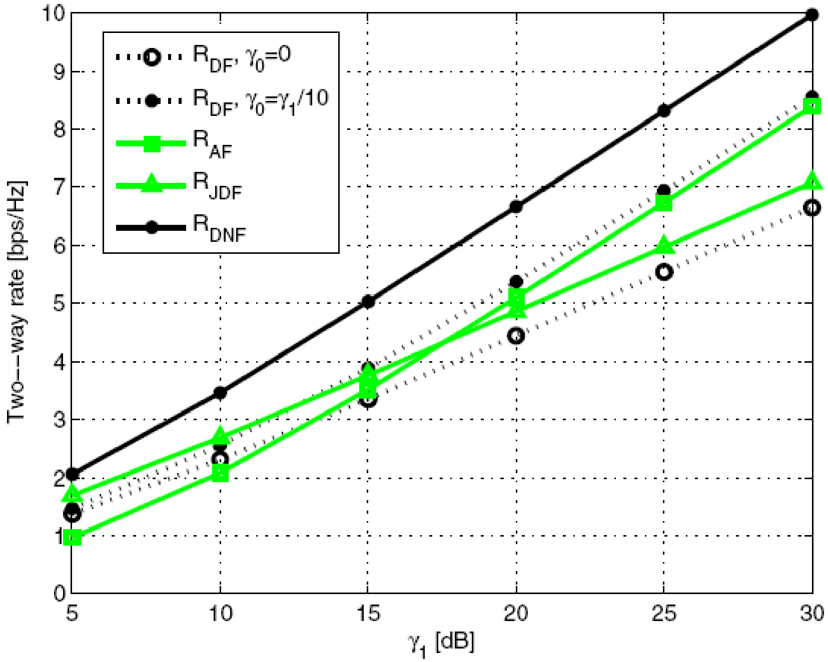
$$\mathcal{L}(\mathbf{y}_B) = \{(\mathbf{x}_A^{(a_1)}, \mathbf{x}_C^{(c_1)}), (\mathbf{x}_A^{(a_2)}, \mathbf{x}_C^{(c_2)}) \cdots (\mathbf{x}_A^{(a_K)}, \mathbf{x}_C^{(c_K)})\}$$



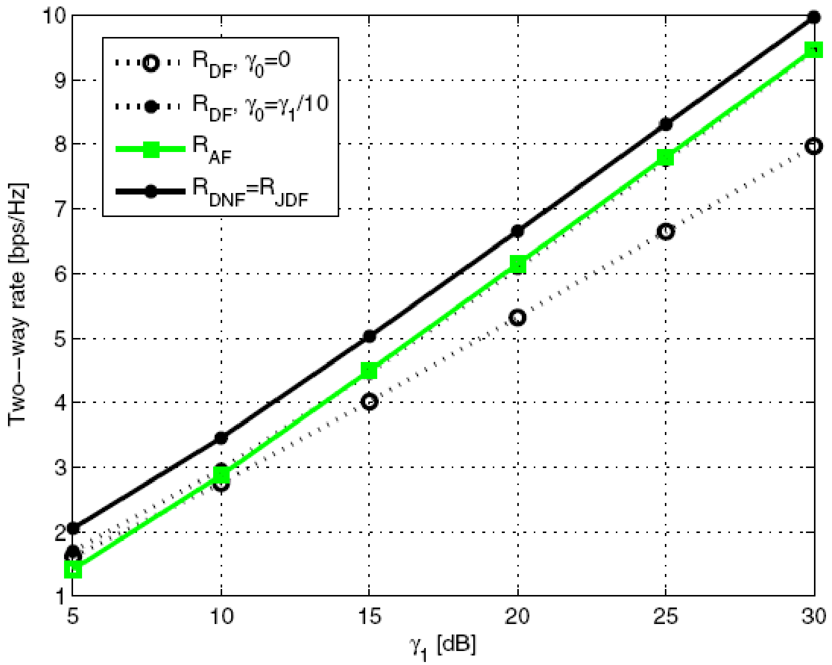
- B broadcasts a quantized version $Q(\mathbf{y}_B)$
- the important question: how many bits are needed to quantize $\mathbf{y}_B$ such that A (C) is able to decode the codeword of C (A) after receiving $Q(\mathbf{y}_B)$?
- upper bound on the rate

$$R^*_{DNF} = C(\gamma_1)$$

$$\gamma_2 = \gamma_1$$



$$\gamma_2 = \gamma_1 + \gamma_1^2$$



conclusions and future work

- two generic types of physical network coding schemes
  - 3-step schemes, 2-step schemes
- we have derived the achievable two-way rates for different schemes for slow fading links
  - under certain operational restrictions
  - for denoise-and-forward (DNF) an upper bound was given
- the interesting points
  - for some SNR configurations, AF is better than JDF
  - for some SNR configurations, DNF and JDF have same rate
- future works
  - proving the achievability of the upper bound for DNF
  - generalization to multiple flows