utilization of superposition coding for achieving spectrally-efficient relaying schemes

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- motivation
- target scenario
- the optimal relaying scheme
- relaying with superposition coding sc-relaying
- sc-relaying for multiple channels
- some extensions
- conclusions and further work

acknowledgements & credits

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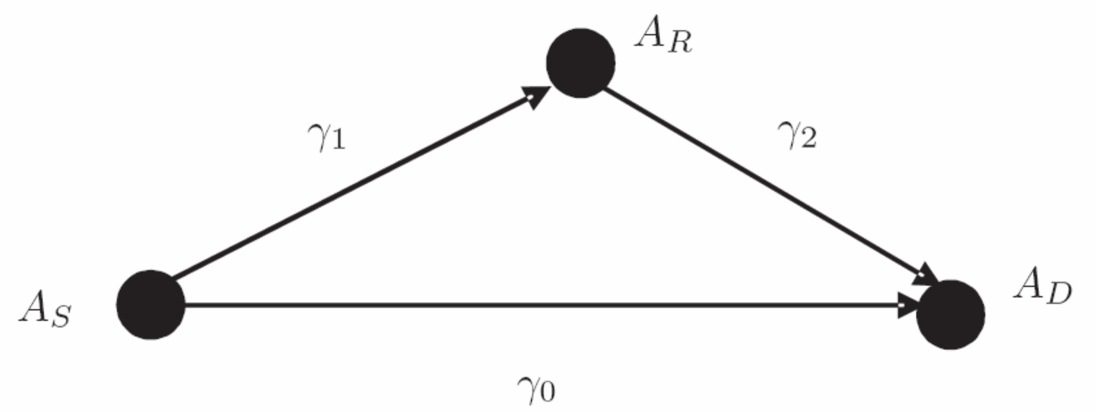
- currently there is a significant interest in (multi-hop, relay, cooperative) transmissions

- however, some of the main concepts are quite old
  - radio-relaying
  - information-theoretic treatment of the relaying (Van Der Meulen, Cover and El Gamal)
  - ad hoc networking is not anymore new as well

- so, why still such a big interest?
  - practical:
    - cheaper deployment for coverage extension and mitigation of the cell edge problems
    - surge of wireless mesh networks, increasing role of wireless sensor networks
    - proliferation of mobile devices
  - academic:
    - accumulated knowledge (e. g. from MIMO) how to use the degrees of freedom in wireless channels
    - recognition that the shared broadcast medium can give rise to novel communication modes

- transmission schemes for wireless relay
  - with superposition coding
  - with wireless network coding
- multi-user OFDMA systems with relays
  - schemes for radio resource allocation
  - cooperative diversity
  - channel estimation
  - interference cancellation, synchronization problems
- measurements and experimental performance assessment



- we are interested in the scenarios with

$$\gamma_1 > \gamma_0 \qquad \gamma_2 > \gamma_0$$

- depending on what the transmitters know about the channels (CSIT) we can make two different studies
  - without CSIT: optimize the outage probability
  - with CSIT: increase the spectral efficiency
- some in-between studies are possible

- half-duplex devices
- number of channels M=1
- bandwidth normalized to 1Hz
  - time measured in number of symbols
  - maximal rate over a link with SNR of γ is

$$C(\gamma) = \log(1+\gamma) \ \text{[bit/s/Hz]}$$

- unless stated otherwise
  - transmitted packets are sufficiently long to allow usage of long codes with virtually no errors
  - AS knows a priori $\gamma_0, \gamma_1, \gamma_2$
  - AR knows a priori γ_2

- transmission in two steps
  - step 1: AS transmits N symbols at a rate R_1, AR decodes everything, AD listens and memorizes
  - step 2: AR transmits $N \cdot R_R \leq N \cdot R_1$ symbols to AD at a rate R_2, AD receives and combines with the reception from step 1 to decode the original information from AS
- overall rate

$$R_{SD} = \frac{N R_1}{N + \frac{N R_R}{R_2}} = \frac{R_1 R_2}{R_2 + R_R}$$

- AR re-codes the information to be transmitted at R_2
  - if it forwards the identical baseband signal as AS, the spectral efficiency is a priori limited to $R_1/2$

optimal relaying (1)

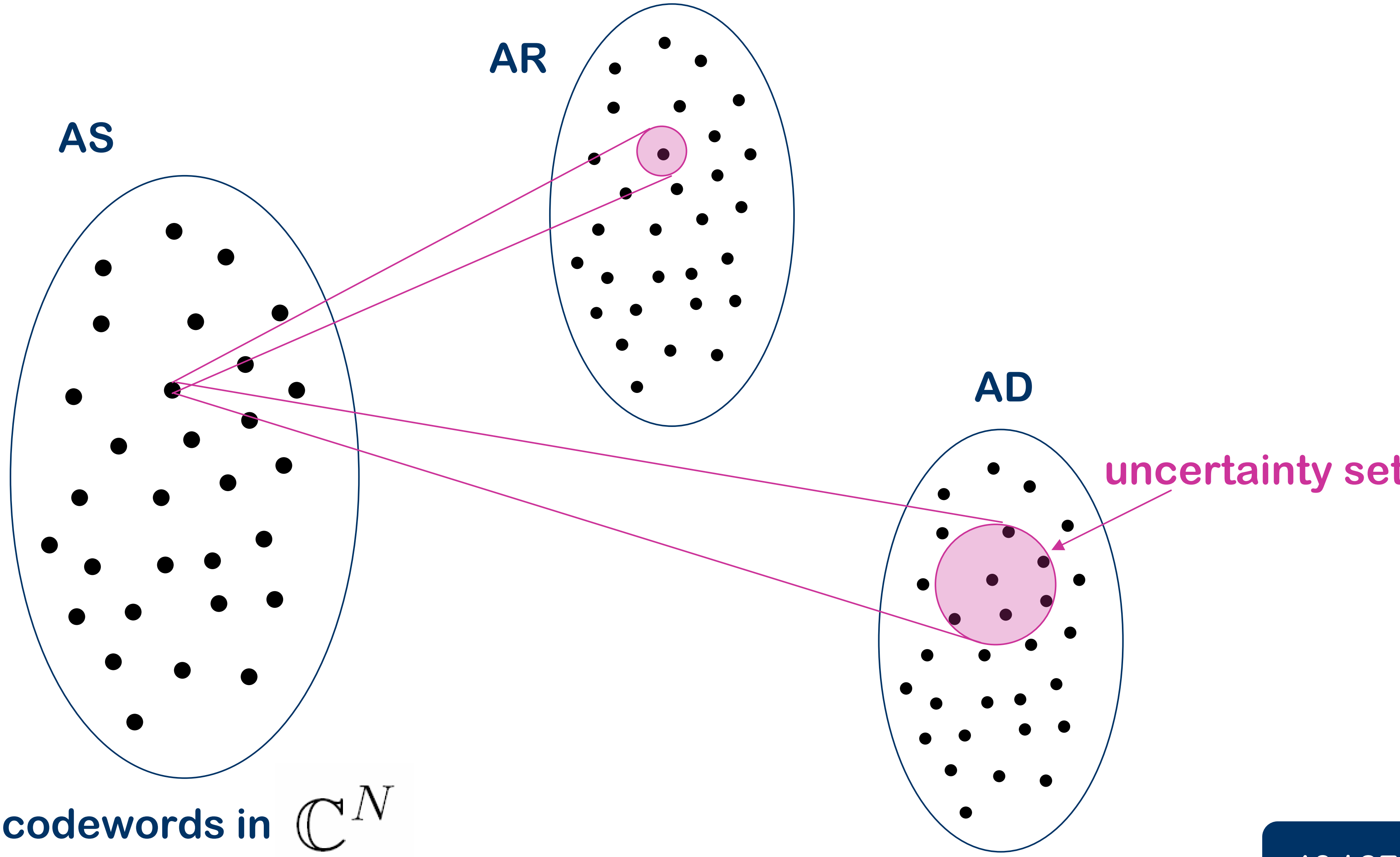
- achieved when R_R is minimal possible
- in Step 1, AS transmits at $R_1 = C(\gamma_1)$ after which

AR receives	AD receives	Uncertainty at AD
$N \cdot C(\gamma_1)$ bits	$N \cdot C(\gamma_0)$ bits	$N \cdot [C(\gamma_1) - C(\gamma_0)]$ bits

- this makes $R_R = C(\gamma_1) - C(\gamma_0)$

$$R_{\text{opt}} = \frac{C(\gamma_1)C(\gamma_2)}{C(\gamma_2) + C(\gamma_1) - C(\gamma_0)}$$

- can be achieved by e. g. random binning



codewords in $\mathbb{C}^N$

- note that AR does not know which are exactly the candidate codewords
- however, for long codewords, AR knows how many codewords on average are in the uncertainty set
- AR randomly partitions the set of all codewords into

$$2^{N[C(\gamma_1)-C(\gamma_0)]}$$

bins, which are a priori known to AD

  - with high probability, each codeword from the uncertainty set belongs to a different bin

- AR sends the bin index to AD

optimal relaying (4)

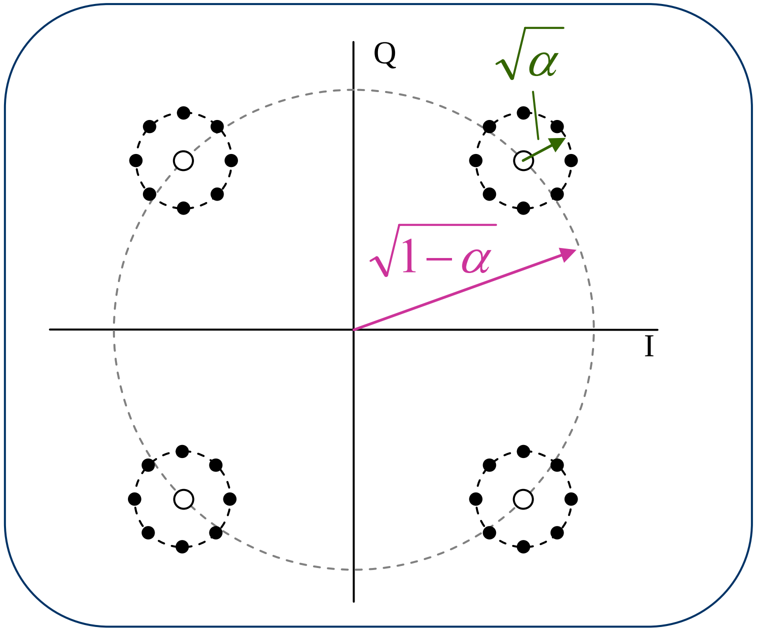
- the presented method is inherently complex
  - practical methods for random binning are under investigation
  - not obvious how it can be transferred to practical, imperfect coded modulations
- how can we approximate this scheme by a more practical approach?

motivating example for superposition coding (1)

- superposition coding
  - two independent messages are added at the baseband
  - the superposed message Xs is an interference for the basic message Xb

$$\sqrt{1-\alpha}x_b + \sqrt{\alpha}x_s \qquad \alpha \in [0,1]$$

- consider uncoded transmission



- AS transmits using this constellation
- AR can send “reliably” to AD using 16-QAM
- AS can send directly to AD “reliably” with QPSK, but not 8-PSK

motivating example for superposition coding (2)

- in this example
  - R_1=2+3=5 bps
  - R_2=4 bps
  - R_R=3 bps

$$R_{sc} = \frac{5 \cdot 4}{4+3} = \frac{20}{7} = 2.86 \text{ bps}$$

- direct transmission would give 2 bps (maybe a little more)

- conventional multi-hop transmission has R_R=5 bps, which results in overall rate of 20/9=2.22 bps

- motivated by this, we propose a relaying scheme based on superposition coding, termed sc-relaying

operation of the sc-relaying scheme (1)

STEP 1

AS transmits

$$\sqrt{1-\alpha}\mathbf{x}_b + \sqrt{\alpha}\mathbf{x}_s$$

AR receives and decodes

$$\mathbf{y}_R = h_1(\sqrt{1-\alpha}\mathbf{x}_b + \sqrt{\alpha}\mathbf{x}_s) + \mathbf{z}_R$$

AD receives and decodes

$$\mathbf{y}_{D1} = h_0(\sqrt{1-\alpha}\mathbf{x}_b + \sqrt{\alpha}\mathbf{x}_s) + \mathbf{z}_{D1}$$

STEP 2

AR recodes and transmits

$$\mathbf{x}_r$$

which contains the information of

$$\mathbf{x}_s$$

AD receives and decodes

$$\mathbf{y}_{D2} = h_2\mathbf{x}_r + \mathbf{z}_{D2}$$

operation of the sc-relaying scheme (2)

- after step 1, AR applies successive interference cancellation: first decodes the basic and then the superposed message

$$R_b \le C\left(\frac{(1-\alpha)\gamma_1}{1+\alpha\gamma_1}\right) = R_{b_1}^U(\alpha)$$

$$R_s \le C(\alpha\gamma_1) = R_s^U(\alpha)$$

$$R_1 = R_b + R_s$$

$$R_R = R_s$$

- after step 2, AD decodes Xr, converts it into Xs, and obtains

$$\mathbf{y}_{D3} = \mathbf{y}_{D1} - h_0\sqrt{\alpha}\mathbf{x}_s = h_0\sqrt{1-\alpha}\mathbf{x}_b + \mathbf{z}_{D1}$$

$$R_b \le C((1-\alpha)\gamma_0) = R_{b_2}^U(\alpha) \qquad R_2 \le C(\gamma_2) = R_2^U(\alpha)$$

optimization of the sc-relaying scheme

- overall rate $$R_{sc} = \frac{(R_b + R_s)R_2}{R_2 + R_s}$$

- optimization problem: given the rate restrictions, choose α that maximizes the achieved rate

- clearly, we set $R_2 = C(\gamma_2)$

- in general, α depends on $\gamma_0, \gamma_1, \gamma_2$

- first note the extreme values
  - α=0 is the direct transmission: $R_s = 0, R_{sc} = R_b = C(\gamma_1)$
  - α=1 is the multi-hop transmission:

$$R_b = 0,\, R_{sc} = \frac{C(\gamma_1)C(\gamma_2)}{C(\gamma_1) + C(\gamma_2)}$$

- restrictions for α

$$R_b \leq C\left(\frac{(1-\alpha)\gamma_1}{1+\alpha\gamma_1}\right) = R_{b_1}^U(\alpha)$$

$$R_b \leq C((1-\alpha)\gamma_0) = R_{b_2}^U(\alpha)$$

$$R_b = \min\left\{R_{b_1}^U, R_{b_2}^U\right\}$$

- a key role is played by $\alpha=\alpha_0$ for which

$$C\left(\frac{(1-\alpha_0)\gamma_1}{1+\alpha_0\gamma_1}\right)=C((1-\alpha_0)\gamma_0)$$

$$\alpha_0=\frac{1}{\gamma_0}-\frac{1}{\gamma_1}$$

- it can be shown that α_0 is optimal when

$$\gamma_2 \geq (1+\gamma_1)^{\frac{\gamma_0(1+\gamma_1)}{\gamma_0(1+\gamma_1)-\gamma_1}}\cdot\left(\frac{\gamma_0}{\gamma_1}\right)-1$$

- using $\alpha=\alpha_0$ makes sc-relaying better than the direct if

$$\gamma_2 \geq (1+\gamma_0)^{\frac{\log_2\left(\frac{\gamma_1}{\gamma_0}\right)}{\log_2\left(\frac{1+\gamma_1}{1+\gamma_0}\right)}}$$

the achieved rate of SC-relaying

- by setting by $\alpha=\alpha_0$ we obtain $$R_{\text{sc}} = \frac{C(\gamma_1)C(\gamma_2)}{C(\gamma_2) + \log_2\left(\frac{\gamma_1}{\gamma_0}\right)}$$

- recall

$$R_{\text{opt}} = \frac{C(\gamma_1)C(\gamma_2)}{C(\gamma_2) + C(\gamma_1) - C(\gamma_0)} = \frac{C(\gamma_1)C(\gamma_2)}{C(\gamma_2) + \log_2\left(\frac{1+\gamma_1}{1+\gamma_0}\right)}$$

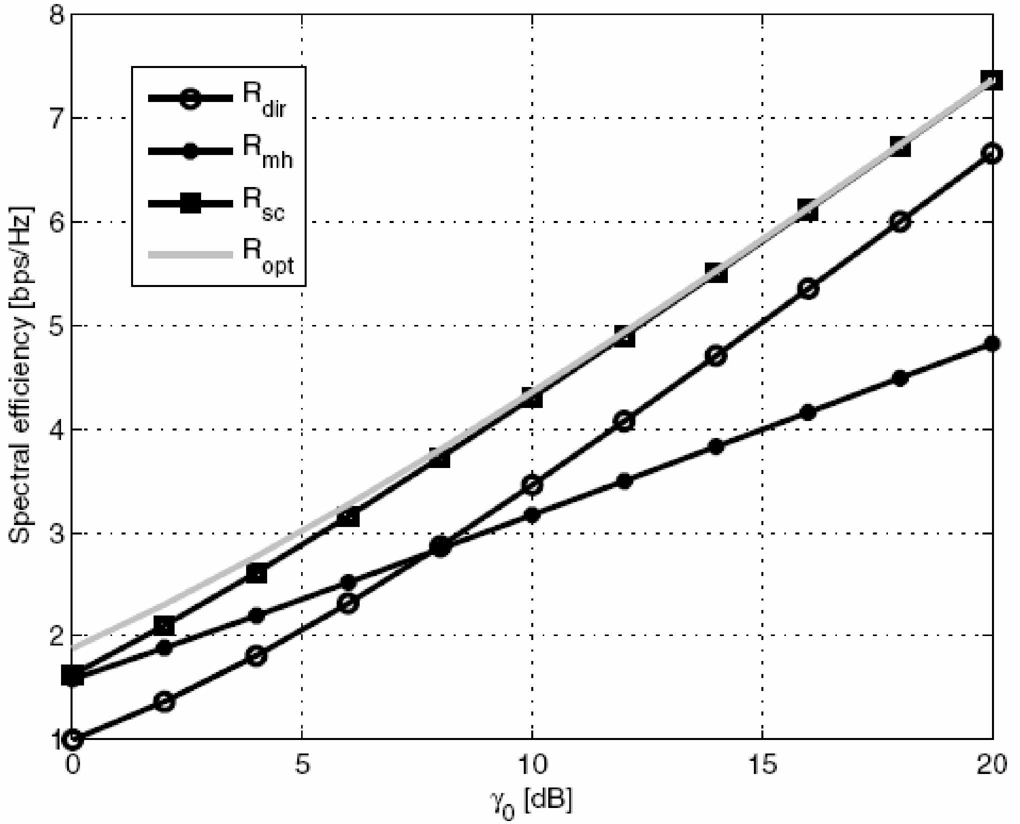
- heuristics when AS does not know the link AR-AD

$$\alpha = \begin{cases} 1 & \text{if } \gamma_0 \leq \frac{\gamma_1}{1+\gamma_1} \\ \frac{1}{\gamma_0} - \frac{1}{\gamma_1} & \text{if } \frac{\gamma_1}{1+\gamma_1} < \gamma_0 < \gamma_1 \\ 0 & \text{if } \gamma_0 \geq \gamma_1 \end{cases}$$

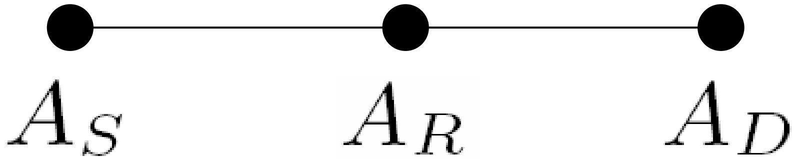
multihop

direct

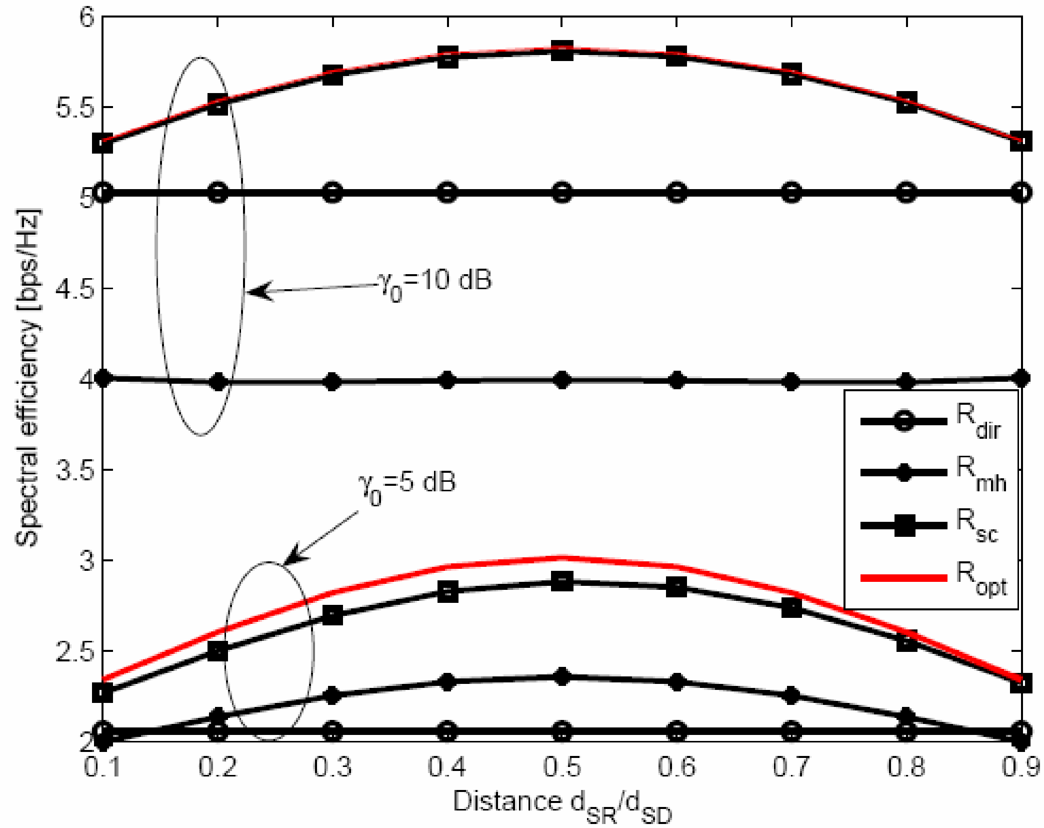
numerical illustration, M=1, static LOS channels (2)

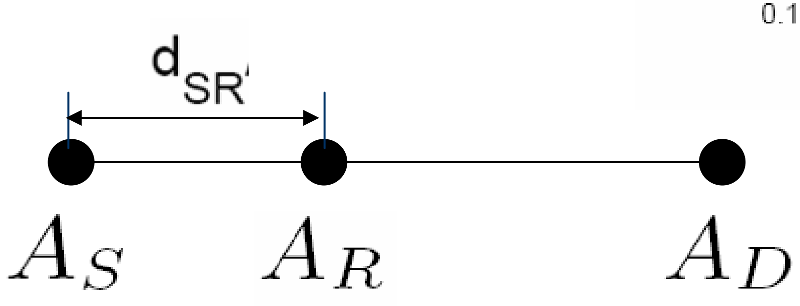


$$\gamma_1 = \gamma_2 = 8\gamma_0$$

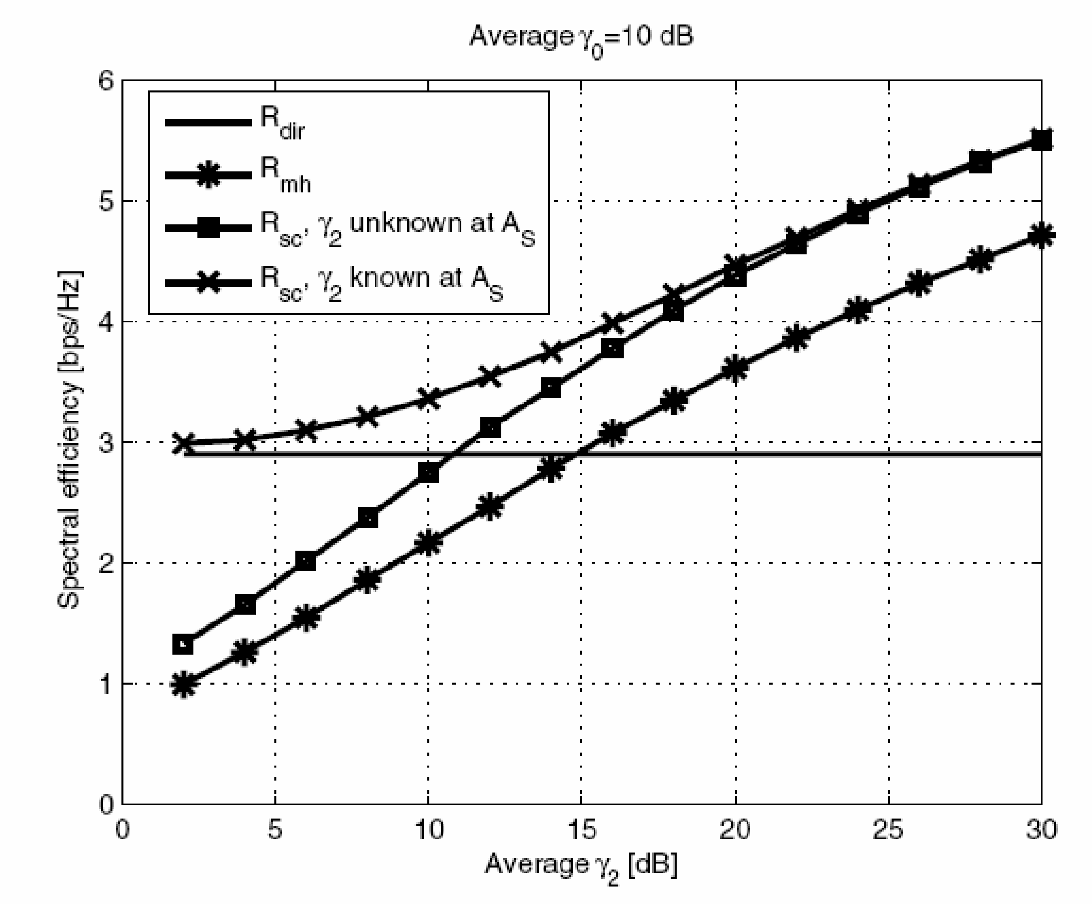


numerical illustration, M=1, static LOS channels (2)





- Rayleigh fading on AS-AD and AR-AD



case of multiple channels M>1

extension of the sc-relaying to multiple channels

- consider OFDM with independently fading subcarriers
  - the m-th subcarrier has $\gamma_{0m}, \gamma_{1m}, \gamma_{2m}$
- a straightforward extension is if we constrain only AR to transmit across all subcarriers in Step 2

example with M=3

subcarrier 1 $\gamma_{11} > \gamma_{01}$	sc-relaying	$\alpha_1 = \frac{1}{\gamma_{01}} - \frac{1}{\gamma_1}$	
subcarrier 2 $\gamma_{12} < \gamma_{02}$	direct	$\alpha_2 = 0$	
subcarrier 3 $\gamma_{13} > \gamma_{03}$	sc-relaying	$\alpha_3 = \frac{1}{\gamma_{03}} - \frac{1}{\gamma_1}$	

step 1: N symbols

data for relaying

step 2:

γ_{21}
γ_{22}
γ_{23}

N_2 symbols

asymptotic spectral efficiency

- we refer to the described scheme as all transmit
- it is interesting to find the spectral efficiency
  - we do it for the case when AS-AR is a LOS link with $\gamma_{1m} = \gamma_1$

total transmitted data

$$D_t = \sum_{\mathcal{M}_0} NC(\gamma_{0m}) + \sum_{\mathcal{M}_1 \cup \mathcal{M}_s} NC(\gamma_1)$$

total relayed data

$$D_r = \sum_{\mathcal{M}_1} NC(\gamma_1) + \sum_{\mathcal{M}_s} N \log_2 \left(\frac{\gamma_1}{\gamma_{0m}} \right)$$

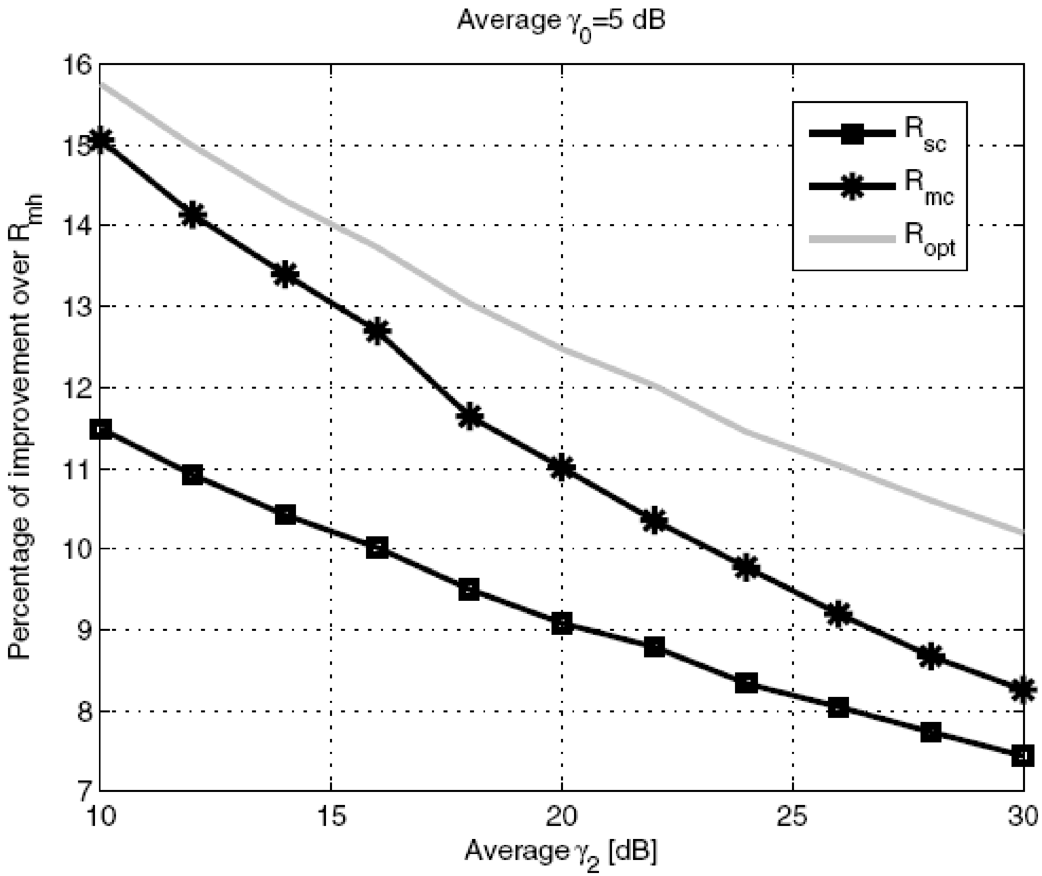
$$\lim_{M\to\infty} \frac{1}{M} D_t = \overline{D}_t \qquad \lim_{M\to\infty} \frac{1}{M} D_r = \overline{D}_r \qquad \lim_{M\to\infty} \frac{1}{M} R_{2t} = \int_0^\infty C(\gamma_2) p_2(\gamma_2) \mathrm{d}\gamma_2 = \overline{R}_{2t}$$

$$\overline{R}_{sc} = \frac{\overline{D}_t}{N + \frac{\overline{D}_r}{\overline{R}_{2t}}}$$

average per-subcarrier spectral efficiency

numerical illustration of the asymptotic efficiency

- asymptotic spectral efficiency is compared to the sc-relaying and optimized relaying over a single channel
- improvement over the pure multi-hop scheme
  - gain(s) on AR-AD are unknown to AS



improvements of the multi-channel transmission (1)

- note that in the step 2 of the all transmit scheme AR may use some subcarriers with low γ_{2m}

- idea: in step 2, AS transmits new data directly to AD by using the subcarriers that have bad γ_{2m} , but good γ_{0m}

subcarrier 4	α_4	relayed data AR-AD
subcarrier 3	α_3	new data AS-AD
subcarrier 2	α_2	relayed data AR-AD
subcarrier 1	α_1	new data AS-AD

step 1
N symbols

step 2
N_2 symbols

time

improvements of the multi-channel transmission (2)

❍ optimization problem

$$\text{maximize } R_{sc} = \frac{\sum_{m=1}^{M} a_m x_m + A}{\sum_{m=1}^{M} b_m x_m + B} \qquad x_m \in \{0, 1\}$$

x_m=1 if the m-th subcarrier in step 2 is used for relaying

$$a_m = d_1 C(\gamma_{2m}) - d_R C(\gamma_{0m}) \qquad A = d_R \sum_{m=1}^{M} C(\gamma_{0m})$$

$$b_m = C(\gamma_{2m}) \qquad B = d_R$$

total fresh data from step 1

$$D_1 = N d_1$$

total to-be-relayed data from step 1

$$D_R = N d_R$$

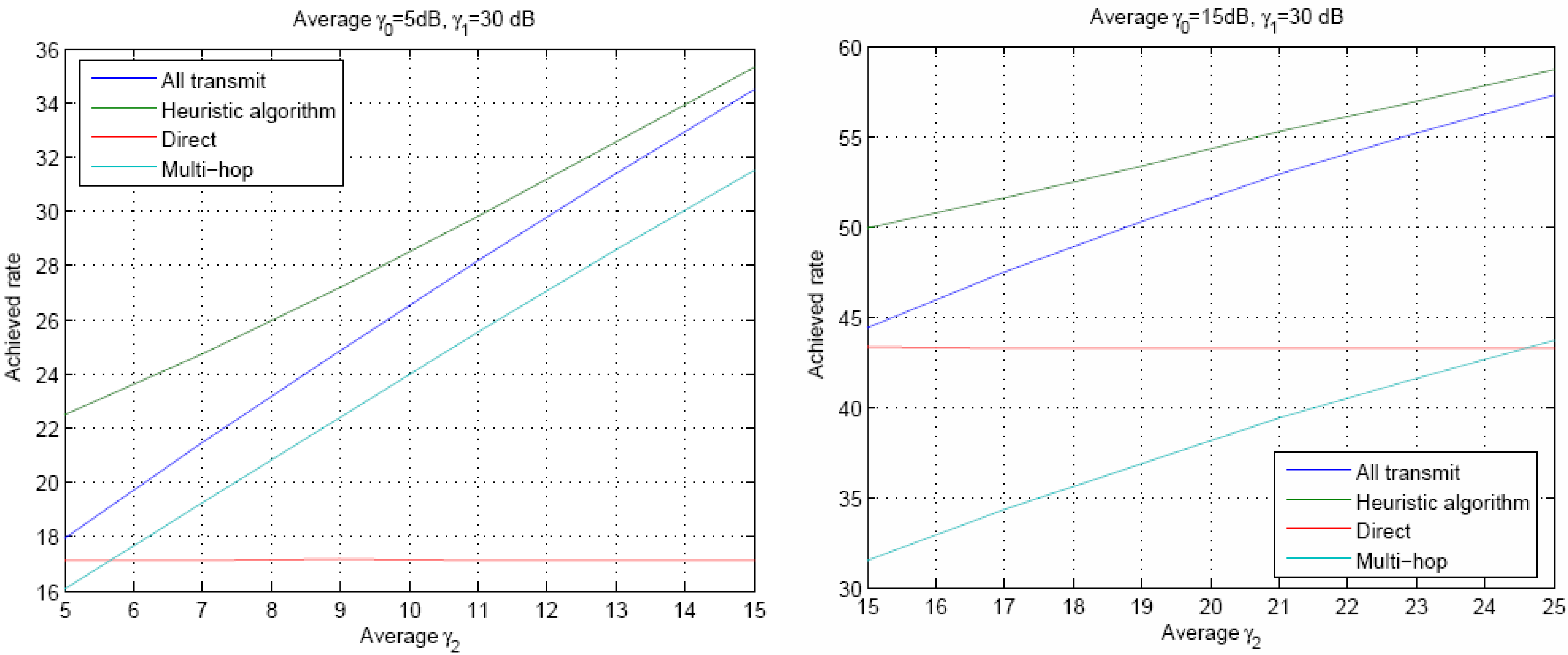
```
initialize R = A/B
for m = 1...M
    c_m = a_m/b_m
c = [c_1  c_2  ...  c_m]
s =sort_descend_indices(c)
inc= true
m = 1; A_0 = A; B_0 = B;
while  m ≤ M  and  inc== true
    A_m = A_{m-1} + a_{s_m}; B_m = B_{m-1} + b_{s_m}; R' = A_m/B_m;
    if  R' > R then  R = R'
    else  inc= false
    m = m + 1
```

- this algorithm is not provably optimal
- an upper bound can be found by finding the optimum with convex relaxation

$$0 \leq x_m \leq 1$$

- however, in all tested cases it gave the same result as the heuristic algorithm

numerical results, M=10 subcarriers



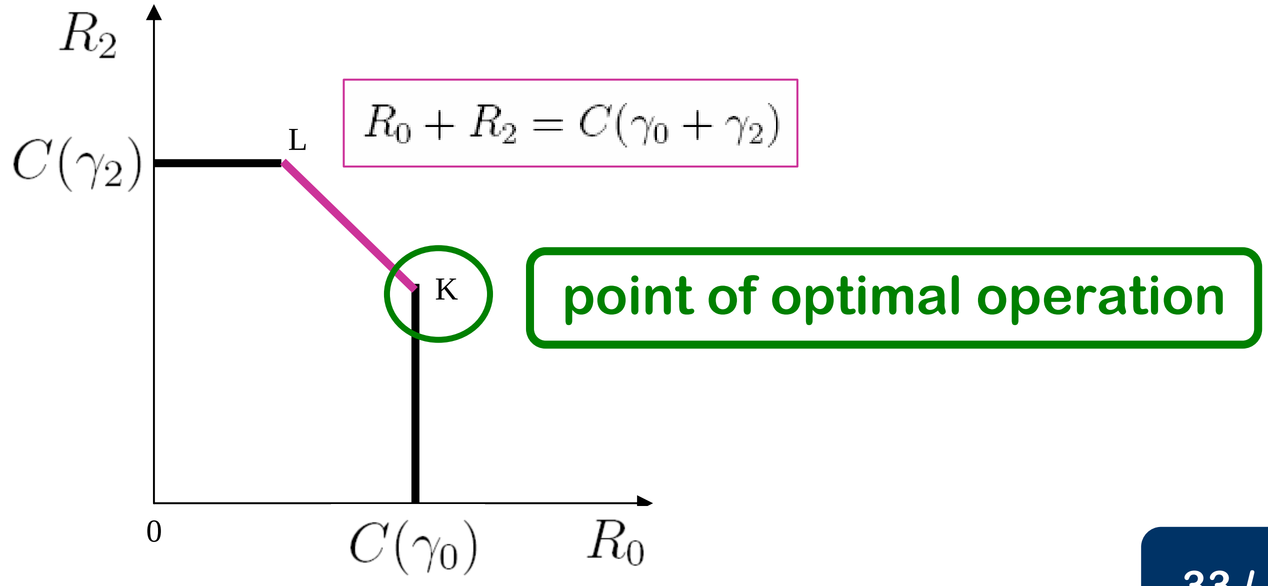
- we have verified that the improvement over the multi-hop transmissions does not change as M grows

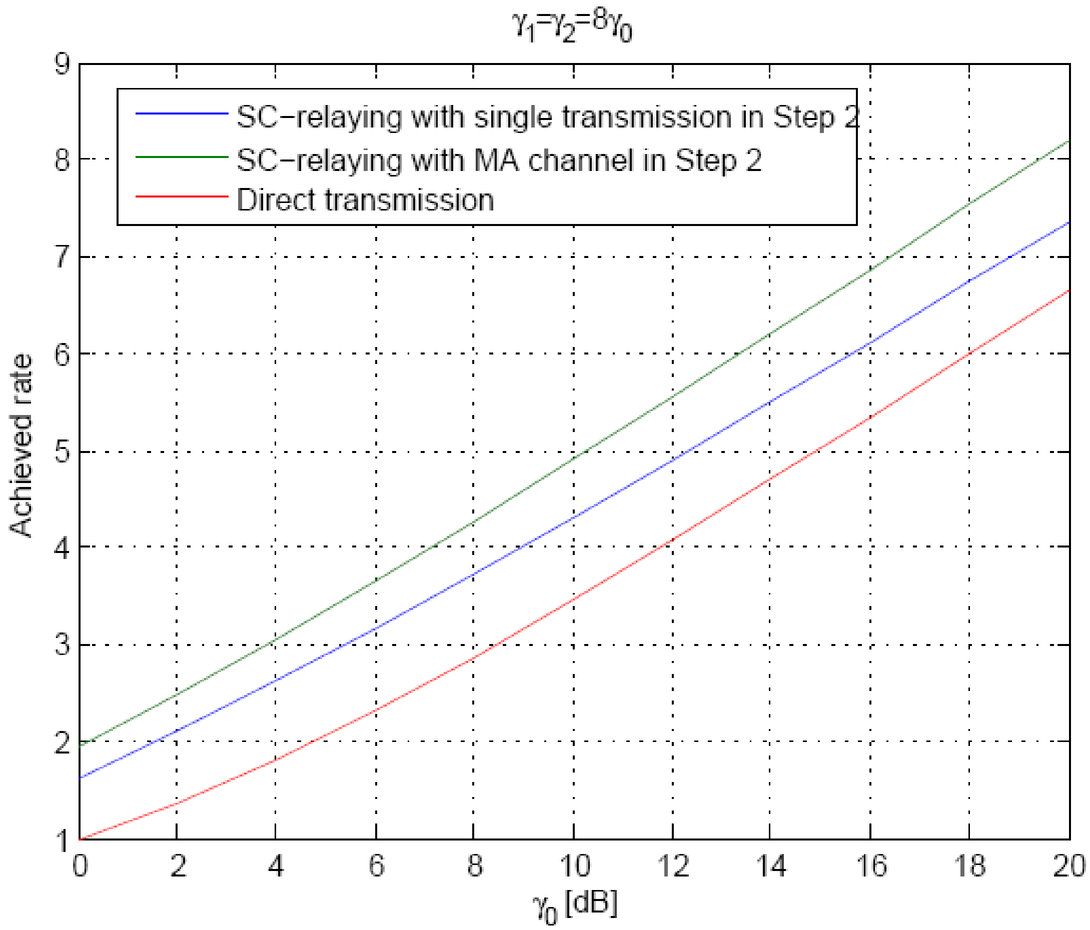
some extensions (for M=1)

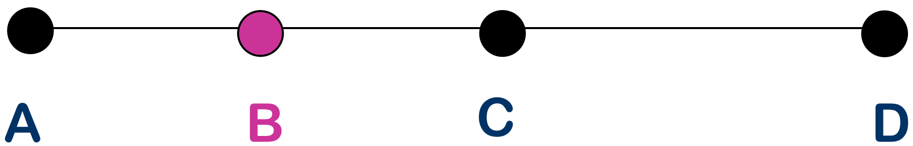
step 2 with multiple access channel

- in step 2 AR relays data at a rate R_0 while AS simultaneously transmits new data at a rate R_2

$$R_{\text{sc--ma}} = \frac{NR_1 + N\frac{R_s}{R_2}R_0}{N + N\frac{R_s}{R_2}} = \frac{R_1R_2 + R_sR_0}{R_2 + R_s}$$



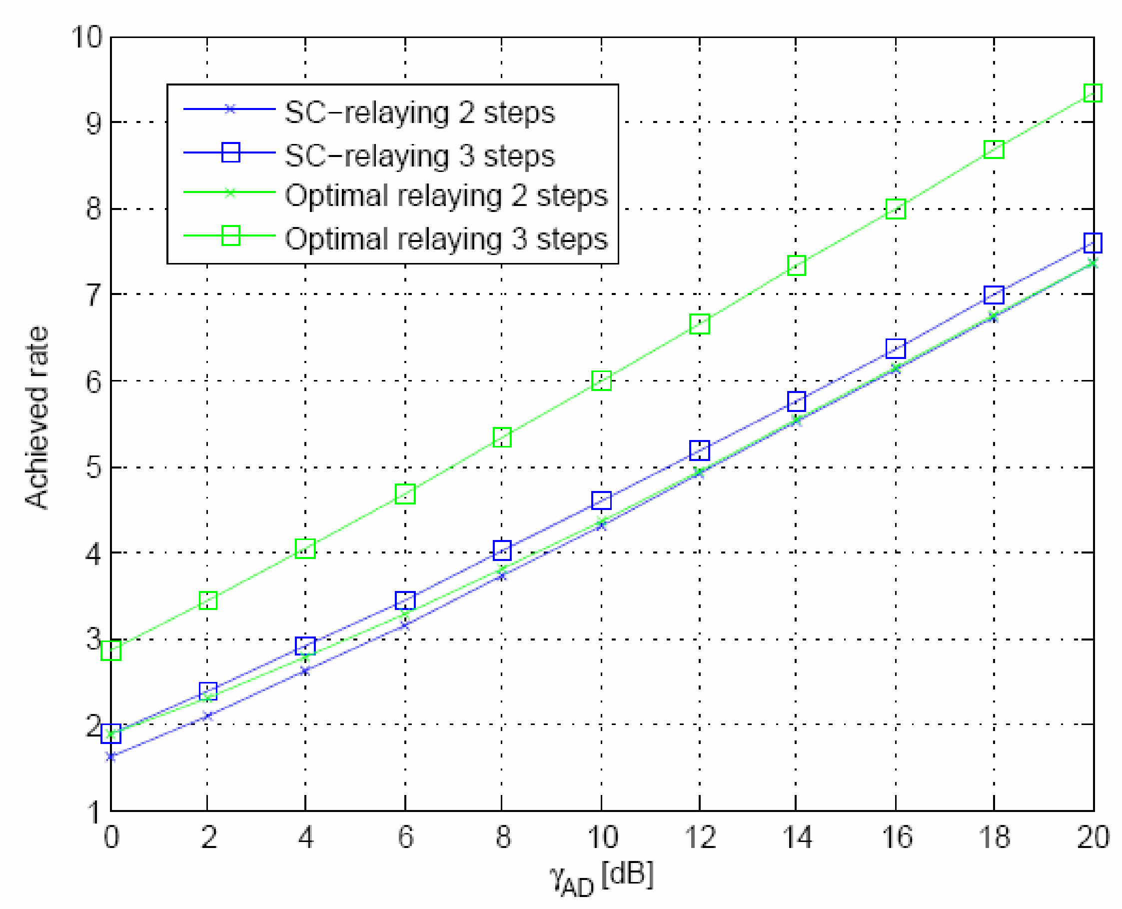

γ1=γ2=8γ0
SC−relaying with single transmission in Step 2
SC−relaying with MA channel in Step 2
Direct transmission
Achieved rate
γ0 [dB]
9
8
7
6
5
4
3
2
1
0
2
4
6
8
10
12
14
16
18
20



- intuitively, the insertion of B should bring increase in the achievable rate
- a possible 3-step relaying
  - step 1: A transmits $\mathbf{x}_A = \sqrt{\alpha_1}\mathbf{x}_1 + \sqrt{\alpha_2}\mathbf{x}_2 + \sqrt{1-\alpha_1-\alpha_2}\mathbf{x}_3$
    B decodes all
  - step 2: B transmits $\mathbf{x}_B = \sqrt{\alpha}\mathbf{x}_{3b} + \sqrt{1-\alpha}\mathbf{x}_{3s}$
    C decodes all
  - step 3: C sends message that contains the information of $\mathbf{x}_2, \mathbf{x}_{3s}$
    D decodes all

- the rates are selected so that the desired decoding operations are possible

numerical results for multiple hops



conclusions and further work

- we have introduced a novel relaying scheme based on superposition coding (sc-relaying)
- sc-relaying:
  - improves the spectral efficiency
  - performs close to the information-theoretic optimal relaying
  - gives also good results in multi-channel systems

- next steps
  - performance evaluation of the sc-relaying with finite modulation/coding schemes
  - efficient extension to multiple hops
  - investigation of these ideas for multi-user systems